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***Corresponding author:**

Email address:

son.thai@hcmut.edu.vn

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A hybrid linear regression and CatBoost model for high-performance nonlinear static analysis of functionally graded plates

Do Thi Thanh Dieu¹, Son Thai^{2,3*}¹Faculty of Management Information Systems, Ho Chi Minh University of Banking, 36 Ton That Dam Street, Saigon Ward, Ho Chi Minh City, Vietnam²Faculty of Civil Engineering, Ho Chi Minh City University of Technology (HCMUT), 268 Ly Thuong Kiet Street, Dien Hong Ward, Ho Chi Minh City, Vietnam³Vietnam National University Ho Chi Minh City, Vo Truong Toan Street, Linh Xuan Ward, Ho Chi Minh City, Vietnam

Abstract: Nonlinear static analysis of functionally graded material (FGM) plates using high-fidelity numerical methods such as Isogeometric Analysis (IGA) is computationally demanding because it requires iterative solution procedures to trace the equilibrium path. Recently, deep learning surrogates (e.g., Long Short-Term Memory, LSTM) have been explored to reduce repeated simulations; however, they can still require nontrivial training effort and careful model tuning. This study proposes a high-performance hybrid surrogate model combining Linear Regression (LR) and CatBoost to predict the nonlinear load-deflection behavior of FGM plates. The equilibrium-path response is formulated as a sequence regression task along pseudo-time load steps and decomposed into two stages: LR captures the global linear trend, while CatBoost learns the nonlinear residual component not represented by LR. The proposed approach is trained and validated using IGA-generated datasets. It is evaluated under different data-availability settings (30%, 50%, and 80% of the equilibrium-path sequence used for training and testing) and assessed in terms of prediction accuracy, hyperparameter sensitivity, and end-to-end computational cost (training + prediction). In our implementation, the combined LR-CatBoost training and prediction time is approximately 0.25 s for the baseline case, excluding the IGA computation required to generate the initially available response segment. Prediction accuracy is high, with mean absolute percentage errors on the order of 10^{-2} to $10^{-1}\%$ depending on the case. These findings indicate that the proposed LR-CatBoost strategy provides a robust and ultra-fast surrogate for near-real-time nonlinear structural analysis of FGM plates.

Keywords: Functionally graded plates, Nonlinear static analysis, Linear regression, Catboost, Hybrid model, Sequence regression.

1. Introduction

Functionally Graded Materials (FGMs) constitute a class of advanced composite materials characterized by a continuous variation of constituent volume fractions and material

properties, commonly transitioning from ceramic-rich regions to metal-rich regions through the thickness [1, 2]. Such smooth gradation can alleviate abrupt property discontinuities that often lead to stress concentration and interlaminar

damage in conventional laminated composites, making FGMs well-suited for high-temperature structural components in aerospace and nuclear engineering. Accurately capturing the mechanical behavior of FGM structures requires robust numerical methods, among which Isogeometric Analysis (IGA) has been widely investigated and successfully applied to plate and shell problems [3–9]. By employing Non-Uniform Rational B-Splines (NURBS) as basis functions, IGA enables an exact geometric representation for many CAD geometries and provides higher-order continuity, which can offer improved accuracy and convergence characteristics (often per degree of freedom) compared with standard polynomial Finite Element Method (FEM) discretizations for a range of problems.

In practical engineering scenarios, FGM plates may experience severe loading conditions that induce large deformations, for which linear theories are no longer adequate. Consequently, nonlinear static analysis becomes essential to evaluate structural stability and to trace the equilibrium path, including potential pre-buckling and post-buckling responses. However, nonlinear static analysis typically requires solving highly nonlinear equilibrium equations using iterative schemes such as the Newton-Raphson method, which can be computationally demanding. The computational burden becomes particularly pronounced in applications requiring repeated analyses, such as design optimization, uncertainty quantification, and, potentially, near-real-time assessment or monitoring tasks [10].

To reduce the cost of high-fidelity simulations, Machine Learning (ML) and Deep Learning (DL) surrogate models have recently attracted increasing attention in computational mechanics [11–17]. In this paradigm, data-driven models learn the mapping from input parameters to structural responses, thereby reducing or bypassing repeated numerical computations once trained. In a previous study, Do et al. [18]

reformulated the nonlinear response prediction of FGM plates as a sequence-learning problem. By treating the incremental load steps as pseudo-time intervals, discrete points along the load-deflection equilibrium path were modeled as an ordered sequence, and Long Short-Term Memory (LSTM) networks were trained to perform one-step-ahead prediction. While such LSTM-based surrogates can substantially accelerate evaluation relative to repeated IGA simulations, their deployment may still involve nontrivial training time (depending on the setting) and hyperparameter/architecture tuning, which can be less convenient when rapid model development and lightweight inference are desired.

In sequence regression and forecasting tasks, hybrid modeling strategies that decompose the target signal into simpler components have long been used to improve predictive performance. The seminal work by Zhang [19] advocated an additive hybrid viewpoint and demonstrated empirically that combining a linear model (ARIMA) with a nonlinear model (ANN) can outperform either model alone on several forecasting problems by allowing the nonlinear learner to correct residual patterns not captured by the linear component. Motivated by this principle, more recent studies in fields such as energy and finance have adopted modern gradient boosting algorithms as powerful nonlinear residual learners. Among them, CatBoost (Categorical Boosting) has emerged as a strong performer due to algorithmic features such as ordered boosting, which is designed to mitigate prediction shift and reduce the risk of target leakage during training, thereby improving generalization [20, 21]. In computational mechanics, the effectiveness of gradient boosting-based surrogates has also been reported. For example, Do and Thai [22] developed a CatBoost-based surrogate model for rapid prediction of free-vibration characteristics of tri-directional functionally graded plates, achieving substantial speed improvements over IGA-based

computations. Related ensemble and interpretable ML approaches have also been successfully applied to a range of transport and construction materials/structures. Prakash and Nguyen [23] optimized an XGBoost model using metaheuristic algorithms and SHAP analysis to predict the maximum load capacity of FRP-confined circular RC columns, demonstrating the effectiveness of boosting-based models for structural capacity assessment. Hoang et al. [24] evaluated several tree-based ensemble learners for Marshall Stability prediction of basalt-fiber-modified asphalt concrete and highlighted the usefulness of SHAP-based interpretation for pavement material assessment. Asteris et al. [25] further developed an interpretable ML pipeline for flexural strength prediction of ultra-high-performance concrete, showing that CatBoost and other boosting models can provide accurate and transparent predictions for nonlinear cementitious-material behavior. Collectively, these studies suggest that boosting and ensemble frameworks can provide accurate, interpretable, and computationally efficient surrogates for complex mechanical behaviors.

Despite these advances, the use of a residual-decomposition hybrid framework specifically tailored to predicting nonlinear static equilibrium paths of FGM plates, where a simple model captures the global trend and a powerful nonlinear learner models the remaining residual response, has been limited in the existing literature to the best of our knowledge. To this end, the present study proposes a hybrid machine learning framework combining Linear Regression (LR) and CatBoost for rapid prediction of the nonlinear behavior of FGM plates. Unlike dynamic problems involving physical time, here the equilibrium path is treated as a sequence evolving along load increments (pseudo-time). In the proposed architecture, the one-step-ahead prediction task is decomposed into two stages: LR is employed to represent the global linear trend of the response evolution, while CatBoost is trained to learn the

nonlinear residuals not captured by LR. The proposed model is trained and validated using datasets generated by IGA and is benchmarked against the LSTM-based approach established in [18].

The present study adopts a time-series forecasting perspective, in which an initially available portion of the response sequence is used to predict the subsequent evolution of the same equilibrium path. For each prescribed value of n , the corresponding IGA-generated sequence is evaluated independently. The observed response history carries information about the combined influence of the physical and loading characteristics of the configuration under consideration, although these parameters are not introduced as explicit model inputs.

The results show that the LR-CatBoost hybrid achieves consistently low prediction errors for the independently evaluated power-law indices and training ratios, while maintaining a lightweight training pipeline and a sub-second end-to-end runtime (training + prediction) in our implementation, indicating its potential as a high-performance surrogate for nonlinear structural analysis of FGM plates.

2. Theoretical background and data generation

This section summarizes the essential theoretical setting of the FGM plate benchmark and explains how the ground-truth data are produced and prepared for surrogate learning. The ground truth consists of nonlinear load-deflection equilibrium paths obtained from geometrically nonlinear static analyses using isogeometric analysis (IGA). For a complete derivation of the nonlinear plate formulation, weak form, and implementation details, readers are referred to our previous work [18].

2.1. Material modeling of functionally graded plates.

We consider a rectangular functionally graded material (FGM) plate of length a , width b , and uniform thickness h , as illustrated in Fig. 1. The

thickness coordinate is denoted by $z \in [-h/2, h/2]$, where the material composition varies smoothly through the thickness, grading from a ceramic-rich region near the top surface ($z = h/2$) to a metal-

rich region near the bottom surface ($z = -h/2$). The volume fractions of ceramic and metal phases are denoted by $V_c(z)$ and $V_m(z)$, satisfying

$$V_c(z) + V_m(z) = 1. \quad (1)$$

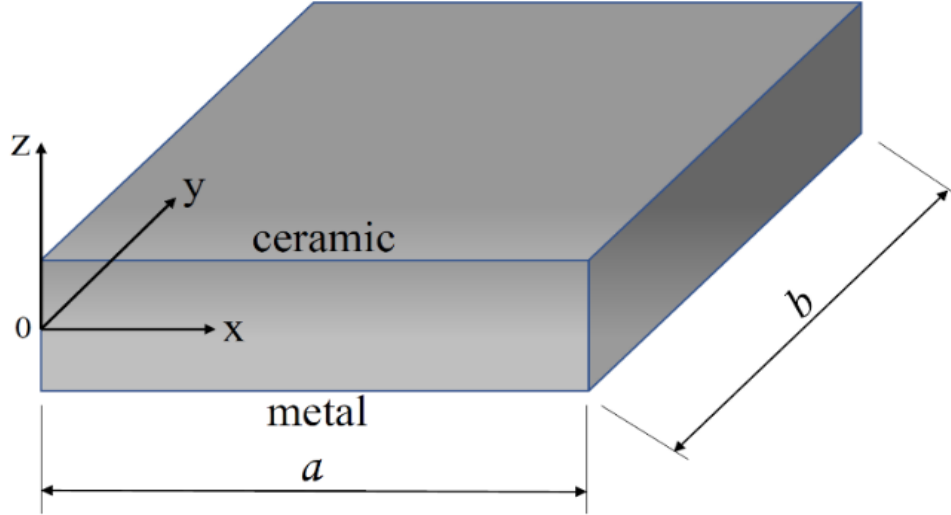


Fig. 1. The functionally graded plate model

The ceramic volume fraction is assumed to follow a power-law distribution,

$$V_c(z) = (0.5 + z/h)^n, n \geq 0 \quad (2)$$

and $V_m(z) = 1 - V_c(z)$. The power-law index n controls the grading profile: $n=0$ yields a fully ceramic plate, while larger n increases the metal content through most of the thickness.

Effective material properties at coordinate z are estimated by the Voigt rule of mixtures. For a generic property $P(z)$,

$$P(z) = P_c V_c(z) + P_m V_m(z), \quad (3)$$

where subscripts c and m refer to ceramic and metal constituents, respectively. In this work, Eq. (3) is applied to compute Young's modulus and Poisson's ratio:

$$\begin{aligned} E(z) &= E_c V_c(z) + E_m V_m(z), \\ \nu(z) &= \nu_c V_c(z) + \nu_m V_m(z). \end{aligned} \quad (4)$$

2.2. Nonlinear static analysis via Isogeometric analysis (IGA)

The nonlinear static responses used as ground truth are generated using the IGA framework. In IGA, spline-based functions (typically NURBS) are used both to represent the geometry and to approximate the displacement

field, which provides smooth higher-order continuity and supports accurate plate/shell analysis. The geometrically nonlinear equilibrium at a given load level can be expressed in a compact symbolic form as

$$\mathbf{K}(\mathbf{u}) \mathbf{u} = \mathbf{F}, \quad (5)$$

where $\mathbf{u}$ is the global displacement vector, $\mathbf{F}$ is the external load vector, and $\mathbf{K}(\mathbf{u})$ denotes a displacement-dependent stiffness operator arising from geometric nonlinearity.

To trace the equilibrium path, the load is applied incrementally. At each load step, the nonlinear system is solved using a Newton-Raphson iterative scheme in an incremental-iterative manner until convergence is achieved. The sequence of converged solutions across load steps forms the nonlinear load-deflection curve (equilibrium path), which is then sampled and stored as discrete data points.

The IGA response database employed in this study was generated from the Al/ZrO₂ functionally graded plate benchmark reported in [9]. Aluminium is characterized by $E_{Al} = 70 \text{ GPa}$, $\nu_{Al} = 0.3$, and $\rho_{Al} = 2707 \text{ kg/m}^3$, whereas the corresponding

properties of zirconia are $E_{ZrO_2}=151\text{GPa}$, $\nu_{ZrO_2}=0.3$, and $\rho_{ZrO_2}=3000\text{ kg/m}^3$. The adopted plate length and thickness are $L=0.2\text{m}$ and $h=0.01\text{m}$, respectively. The plate is subjected to the movable-edge simply supported condition SSSS1, expressed as $v_0=w_0=\beta_y=0$ at $x=0,L$ and $u_0=w_0=\beta_x=0$ at $y=0,W$. A uniform transverse pressure is increased incrementally through 200 load steps until $f_z=-10^7\text{ N/m}^2$, yielding an equilibrium-path sequence containing 200 response values. Seven material-gradation cases are examined, corresponding to $n=0, 0.5, 1, 2, 5, 10$, and ∞ . Following [9], the effective properties are evaluated through the power-law volume-fraction model and the Voigt mixture rule. The plate formulation employs generalized shear deformation theory together with von Kármán nonlinear kinematics, and the resulting nonlinear equations are solved using an incremental Newton–Raphson procedure within the IGA framework.

2.3. Data preparation and learning setup

Ground-truth equilibrium paths: For each material grading case (specified by the power-law index), a nonlinear IGA simulation is performed to produce a load-deflection equilibrium path. Each equilibrium path is sampled at a fixed number of load increments (200 points in this study), resulting in an ordered sequence

$$\{y_1, y_2, \dots, y_N\}, N=200, \quad (6)$$

where y_t denotes the recorded (non-dimensional) deflection value at load step t . Although the response is static, the samples are naturally ordered along load increments; this ordering enables formulating the surrogate problem as sequence prediction along the equilibrium path.

Normalization: Before constructing learning pairs, the raw sequence is normalized using min-max scaling into a bounded interval $[0.3, 0.7]$:

$$\tilde{y}_t = 0.3 + \frac{y_t - y_{\min}}{y_{\max} - y_{\min}} (0.4), \quad (7)$$

The interval $[0.3, 0.7]$ was adopted as a bounded preprocessing convention to place the response sequences on a consistent numerical scale. This choice is not related to activation saturation, since the LR-CatBoost framework does not employ activation functions. No specific performance advantage is attributed to this interval for the CatBoost component, because positive affine scaling preserves the ordering of the numerical input values used to determine tree splits. Therefore, the selected interval is regarded as an implementation choice rather than an essential requirement of the proposed framework.

In our implementation, the scaler parameters $(y_{\min}, y_{\max})$ are fitted on the full sequence and then applied consistently to the training, testing, and prediction segments. After model inference, predictions are inverse-transformed back to the original scale to compute error metrics.

One-step-ahead supervised pairs (look-back = 1): A one-step-ahead setting with look-back length equal to one is adopted. Each input sample uses the previous step value to predict the next step value:

$$x_t = \tilde{y}_t, y_t = \tilde{y}_{t+1}, t=1, \dots, N-1. \quad (8)$$

This produces a supervised dataset of $(N-1)$ input-output pairs for each equilibrium path. The response sequence corresponding to each prescribed value of n is processed independently. The initial portion of each sequence is used to develop the LR-CatBoost model for forecasting the subsequent evolution of the associated equilibrium path. The look-back value $p=1$ is adopted as a parsimonious formulation to evaluate whether the immediately preceding response is sufficient for forecasting the smooth equilibrium paths considered in this study. Larger look-back windows were not examined in the present assessment. Such windows may provide additional information about the local evolution of the response, although explicit first- and second-order differences would be required to represent slope- and curvature-

related features directly.

Data splitting (30%/50%/80% with 80/20 train-test): To examine model robustness under different data availability levels, three training-data ratios are considered:

$$r \in \{0.3, 0.5, 0.8\}. \quad (9)$$

For a given ratio r , the first $N_r = \lfloor rN \rfloor$ points of the normalized sequence are reserved for model development (training + testing), and the remaining $N - N_r$ points are used for forecasting (prediction). In the present study, the initially available response segment is obtained from the IGA solution and is used to develop the forecasting model for the subsequent portion of the same equilibrium path. The ratios of 30%, 50%, and 80% represent alternative data-availability scenarios and should not be interpreted as universal minimum requirements. More generally, the same sequence-forecasting formulation may be applied when an initial segment of the structural response is

available from measurements or prior numerical analysis, and the objective is to estimate its subsequent evolution. Within the model-development segment, an 80/20 split is applied:

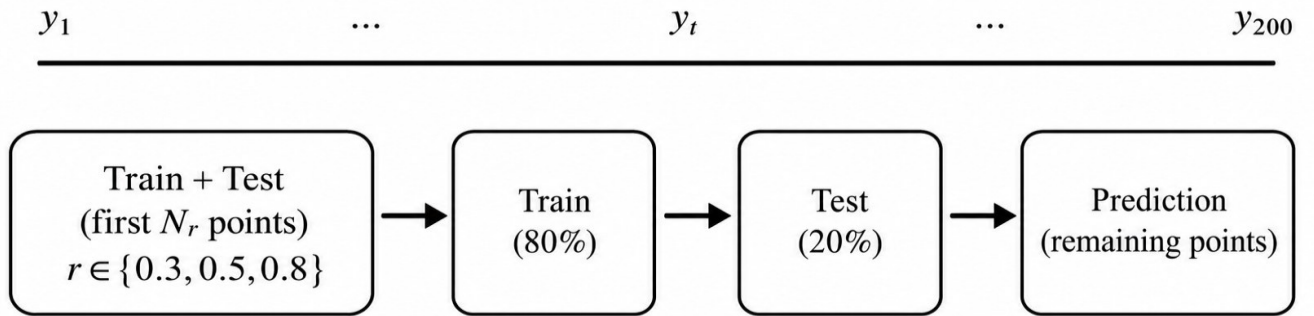
Training segment: first $\lfloor 0.8N_r \rfloor$ points (converted into one-step pairs),

Testing segment: remaining $N_r - \lfloor 0.8N_r \rfloor$ points (converted into one-step pairs),

Prediction segment: the rest of the sequence from step $N_r + 1$ to N , used to evaluate forecasting performance.

Accordingly, three error quantities are reported: training error (on the training pairs), testing error (on the testing pairs), and prediction/forecast error (on the prediction segment). All errors are computed after inverse transformation to the original response scale. The normalization, one-step-ahead pair construction, and data partitioning scheme are illustrated in Fig. 2.

Equilibrium-path sequence ($N = 200$):



Supervised pairs (look-back = 1): $x_t = \bar{y}_t \rightarrow y_t = \bar{y}_{t+1}$ (Min-max scaling to $[0.3, 0.7]$)

Fig. 2. Data normalization, one-step-ahead pair construction (look-back = 1), and 30/50/80 data partitioning

3. Proposed hybrid learning framework

This section presents the proposed hybrid learning framework, which constitutes the core contribution of this study. The main idea is to decompose the one-step-ahead prediction along the nonlinear equilibrium path into two parts: (i) a simple linear model that captures the dominant global trend, and (ii) a nonlinear learner that

corrects the remaining residual component. The resulting LR-CatBoost surrogate is designed to be accurate, lightweight, and fast to train and deploy.

3.1. Problem formulation: sequence regression along load steps (pseudo-time)

In this study, the nonlinear equilibrium path obtained from incremental loading is treated as an ordered response sequence indexed by a pseudo-

time (load-step) variable t . This index represents the step order of the numerical procedure and does not correspond to physical time. Let y_t denote the (normalized) non-dimensional central deflection at pseudo-time step t , for $t = 1, 2, \dots, N$.

With a look-back of one ($p = 1$), we formulate a one-step-ahead sequence regression problem where the input is the current response and the target is the next-step response:

$$x_t = y_t, \quad \hat{y}_t^{+1} = f(x_t). \quad (10)$$

Accordingly, the supervised dataset is constructed from input-target pairs along the load-step sequence:

$$D = \{(x_t, y_t^{+1}) | t = 1, 2, \dots, N-1\} \quad (11)$$

Here, $f(\cdot)$ denotes the learned mapping (LR-CatBoost in this work). After training, the model is used to predict the next response point sequentially to reconstruct the equilibrium-path progression over the testing and prediction segments

3.2. Linear regression as a global trend learner

As the first stage, a linear regression (LR) model is used to represent the dominant global trend of the one-step-ahead mapping. Given a scalar input x , the linear predictor is written as

$$\hat{y}_{lin} = wx + b, \quad (12)$$

where w is the regression coefficient and b is the bias term. The parameters (w, b) are obtained by minimizing the squared error on the training set.

The role of LR in the proposed framework is not to fully approximate the nonlinear relationship, but to provide a fast, stable estimate of the global trend. This decomposition is beneficial because the remaining discrepancy (residual) typically has smaller magnitude and can be learned more effectively by a nonlinear model in the second stage.

It should be noted that the linear component does not impose an explicit monotonicity constraint on the predicted response, since nonlinear deviations from the dominant trend are represented by the residual learner. However, for

equilibrium paths containing pronounced turning points or multiple branches, a single preceding response value may not provide sufficient information to uniquely determine the subsequent state. The performance of the present $p=1$ formulation for such responses remains to be investigated.

3.3. CatBoost as a residual learner

The second stage employs CatBoost, a gradient boosting decision-tree algorithm, to model the residual component that is not captured by the linear trend. Gradient boosting constructs an additive model by sequentially fitting weak learners (decision trees) to reduce the current prediction error. CatBoost introduces algorithmic mechanisms such as ordered boosting, which is designed to mitigate prediction shift caused by target leakage during boosting iterations, thereby improving generalization.

In the proposed framework, CatBoost does not learn the full response y directly. Instead, it learns the residuals with respect to the LR prediction. This “residual learning” setup typically reduces the complexity of the target function, enabling robust performance even with shallow trees and a modest number of boosting iterations.

3.4. LR-CatBoost hybrid architecture and training

Two-stage prediction model: Let denote the LR prediction from Eq. (12). The residual at each training sample is defined as

$$r = y - \hat{y}_{lin}, \quad (13)$$

and CatBoost is trained to approximate the mapping $x \mapsto r$. Denoting the CatBoost residual prediction by $\hat{y}_{res} \equiv \hat{r}$, the final hybrid prediction is computed by a simple additive combination:

$$\hat{y}_{final} = \hat{y}_{lin} + \hat{y}_{res}. \quad (14)$$

This architecture explicitly separates the global trend and the nonlinear correction, which improves both interpretability and computational efficiency.

The training process consists of two

consecutive phases:

- Stage 1 (Trend fitting):

1. Train LR using the training pairs $\{(x_i, y_i)\}$.

2. Compute trend predictions $\hat{y}_{lin,i}$ and

residual targets $r_i = y_i - \hat{y}_{lin,i}$.

- Stage 2 (Residual fitting):

3. Train CatBoost using the same inputs x_i but with residual labels r_i .

At inference time, given a new input x , the

model outputs $\hat{y}_{final}$ via Eq. (14) and then applies inverse normalization to obtain predictions in the original response scale.

The overall pipeline, from IGA-generated equilibrium paths to LR-CatBoost training and inference, is summarized in Fig. 3, which highlights the two-stage training procedure: LR is fitted first, and CatBoost is subsequently trained on the residuals.

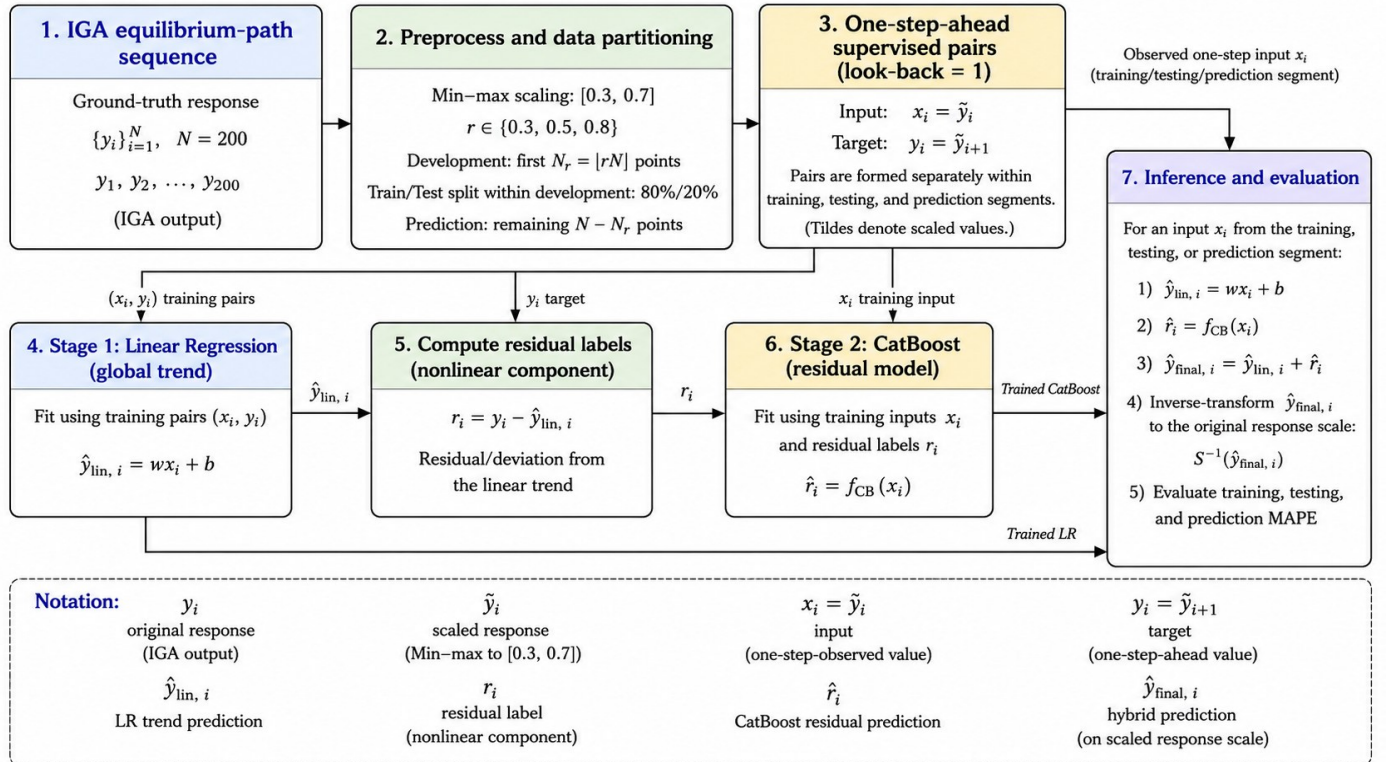


Fig. 3. Workflow of the proposed LR-CatBoost hybrid model

Algorithmic summary: Algorithm 1: LR-CatBoost hybrid training and prediction

1) Input: normalized supervised pairs $\{(x_i, y_i)\}$, with look-back = 1.

2) Train LR on $\{(x_i, y_i)\} \rightarrow$ obtain $\hat{y}_{lin,i}$.

3) Compute residual labels $r_i = y_i - \hat{y}_{lin,i}$.

4) Train CatBoost on $\{(x_i, r_i)\}$.

5) Prediction: for a new x , compute

$$\hat{y}_{final} = \hat{y}_{lin}(x) + \hat{r}(x).$$

6) Inverse-transform $\hat{y}_{final}$ to the original scale for evaluation.

4. Results and discussion

All experiments were conducted in Python

3.7 on a Windows 11 (64-bit) laptop equipped with an Intel® Core™ i7-8550U CPU @ 1.80-2.00 GHz and 12.0 GB RAM. The computational times reported in this section include LR-CatBoost model training and prediction only. The IGA computation required to generate the initially available portion of the equilibrium path is not included. Unless otherwise stated, the CatBoost residual learner was trained with iterations = 1000. To obtain a stable and reusable configuration, the proposed LR-CatBoost framework was first calibrated on the baseline dataset ($n = 0$) with 50% of the sequence allocated to model development (training + testing) and the remainder reserved for prediction.

Following the data preparation strategy described in Section 2 (look-back, min-max scaling), 50% of the sequence was allocated to the

model-development portion (training + testing), and the remaining part was reserved for prediction evaluation.

Table 1. CatBoost hyperparameter screening for the baseline case ($n = 0$, 50% train+test).

Depth	Evaluation		Learning rate				
			0.01	0.05	0.1	0.2	0.3
1	MAPE (%)	Training	0.0025	0.0003	0.0003	0.0003	0.0003
		Testing	0.0082	0.0065	0.0065	0.0064	0.0065
		Prediction	0.0359	0.0348	0.0348	0.0347	0.0347
	Computation time (s)		0.20	0.27	0.23	0.25	0.27
2	MAPE (%)	Training	0.0006	0.0001	0.0001	0.0000	0.0000
		Testing	0.0069	0.0064	0.0064	0.0064	0.0064
		Prediction	0.0350	0.0347	0.0347	0.0347	0.0347
	Computation time (s)		0.30	0.31	0.30	0.30	0.28
3	MAPE (%)	Training	0.0005	0.0001	0.0000	0.0000	0.0000
		Testing	0.0068	0.0064	0.0064	0.0064	0.0064
		Prediction	0.0350	0.0347	0.0347	0.0347	0.0347
	Computation time (s)		0.37	0.34	0.37	0.34	0.34
4	MAPE (%)	Training	0.0004	0.0001	0.0000	0.0000	0.0000
		Testing	0.0067	0.0064	0.0064	0.0064	0.0064
		Prediction	0.0349	0.0347	0.0347	0.0347	0.0347
	Computation time (s)		0.45	0.47	0.45	0.47	0.45

A grid search was conducted over two key CatBoost hyperparameters: tree depth (depth=1,2,3,4) and learning rate ($\eta=0.01,0.05,0.1,0.2,0.3$). For each configuration, three mean absolute percentage errors (MAPE) were reported-training, testing, and prediction-together with the total computational time. The detailed results are summarized in Table 1 (values in bold indicate the best-performing configuration chosen for later experiments).

Linear Regression (LR) stage: Using look-back = 1, LR is fitted by ordinary least squares on the scalar pairs $(x_t, y_t) = (\tilde{y}_t, \tilde{y}_{t+1})$ to capture the global linear trend, i.e., $\hat{y}_{lin} = wx + b$. Residuals $r_t = y_t - \hat{y}_{lin,t}$ are then used to train CatBoost.

Overall, the results show that increasing model complexity (larger depth) yields negligible improvement in testing/prediction MAPE once a sufficiently expressive residual learner is reached. In particular, several settings achieved nearly

identical testing accuracy ($\approx 0.0064\%$) and prediction accuracy ($\approx 0.0347\%$), while computational time increased with depth. This indicates that, for the considered one-step-ahead surrogate, the residual component is relatively easy to learn and does not require deep trees.

Based on the best trade-off between accuracy and efficiency (highlighted in Table 1), the configuration depth = 1 and learning rate = 0.2 was selected as the optimal setting. This setting achieved the lowest testing MAPE (0.0064%) and lowest prediction MAPE (0.0347%) among the tested candidates, while maintaining a very small computational time (0.25 s) compared with deeper trees. Although deeper models (e.g., depth =2-4) can also match the same prediction MAPE, they require more computation (typically ≥ 0.28 -0.47 s in this experiment) without providing consistent accuracy gains, and may increase the risk of overfitting to the training residuals.

Table 2. Comparison between LSTM and LR-CatBoost for the FG plate (50% and 80% train+test).

Power index (n)	Training set size (%)	Method	Mean absolute percentage error (%)			Computation time (s)
			Training	Testing	Prediction	
0	50%	LSTM	0.0516	0.0621	0.0456	142.55
		LR-CatBoost	0.0003	0.0064	0.0347	0.25
	80%	LSTM	0.2911	0.2562	0.1962	208.57
		LR-CatBoost	0.0005	0.0073	0.0202	0.27
0.5	50%	LSTM	0.0450	0.0291	0.0873	148.44
		LR-CatBoost	0.0005	0.0095	0.0463	0.23
	80%	LSTM	0.0631	0.0520	0.0239	215.40
		LR-CatBoost	0.0007	0.0088	0.0230	0.27
1	50%	LSTM	0.0372	0.0094	0.1375	158.46
		LR-CatBoost	0.0006	0.0108	0.0504	0.22
	80%	LSTM	0.0484	0.0437	0.1259	211.36
		LR-CatBoost	0.0007	0.0091	0.0229	0.25
2	50%	LSTM	0.0250	0.0061	0.1395	144.06
		LR-CatBoost	0.0006	0.0116	0.0524	0.25
	80%	LSTM	0.2120	0.2264	0.3175	205.70
		LR-CatBoost	0.0009	0.0090	0.0223	0.25
5	50%	LSTM	0.0239	0.0304	0.1707	149.88
		LR-CatBoost	0.0007	0.0121	0.0533	0.22
	80%	LSTM	0.3980	0.0089	0.0551	206.03
		LR-CatBoost	0.0010	0.0089	0.0218	0.25
10	50%	LSTM	0.0136	0.0233	0.1697	144.26
		LR-CatBoost	0.0008	0.0126	0.0543	0.22
	80%	LSTM	0.5757	0.1242	0.0363	206.99
		LR-CatBoost	0.0009	0.0087	0.0208	0.24
inf	50%	LSTM	0.0231	0.0113	0.1722	145.27
		LR-CatBoost	0.0008	0.0144	0.0536	0.22
	80%	LSTM	0.1597	0.0511	0.1204	217.92
		LR-CatBoost	0.0012	0.0066	0.0133	0.25

Consequently, the remainder of this study adopts iterations = 1000, depth = 1, and learning rate = 0.2 for the CatBoost residual learner, while keeping the linear regression stage unchanged. This choice ensures a lightweight surrogate that preserves high accuracy while minimizing runtime, which is particularly desirable for repeated evaluations across different material grading indices and data availability settings.

Using the selected LR-CatBoost configuration (iterations = 1000, depth = 1, learning

rate = 0.2), we next evaluated the surrogate across different material gradings (power-law index n) and two data-availability settings (50% and 80% of the sequence used for model development). Table 2 reports MAPE on the training, testing, and prediction segments together with the total runtime measured on the computer platform described in the above platform description.

For the prediction segment, which constitutes the actual forecasting horizon, LR-CatBoost yields lower MAPE than LSTM for all

power-law indices and both data splits (0.0133–0.0543% versus 0.0239–0.3175%), while the total runtime is reduced from ~140–218 s to ~0.22–0.27 s. In the testing segment, the comparison was mixed: LR-CatBoost gives lower testing MAPE in most cases, whereas LSTM attains lower testing MAPE for $n = 1, 2$ and ∞ in the 50% setting (0.0094%, 0.0061% and 0.0113% versus 0.0108%,

0.0116% and 0.0144%). Both models give an identical testing MAPE of 0.0089% for $n = 5$ in the 80% setting. Since all testing errors are of the order of 0.01%, these differences are small in practical terms. In addition, the main accuracy advantage of LR-CatBoost appears in the prediction segment, where it extrapolates beyond the data used for training and testing.

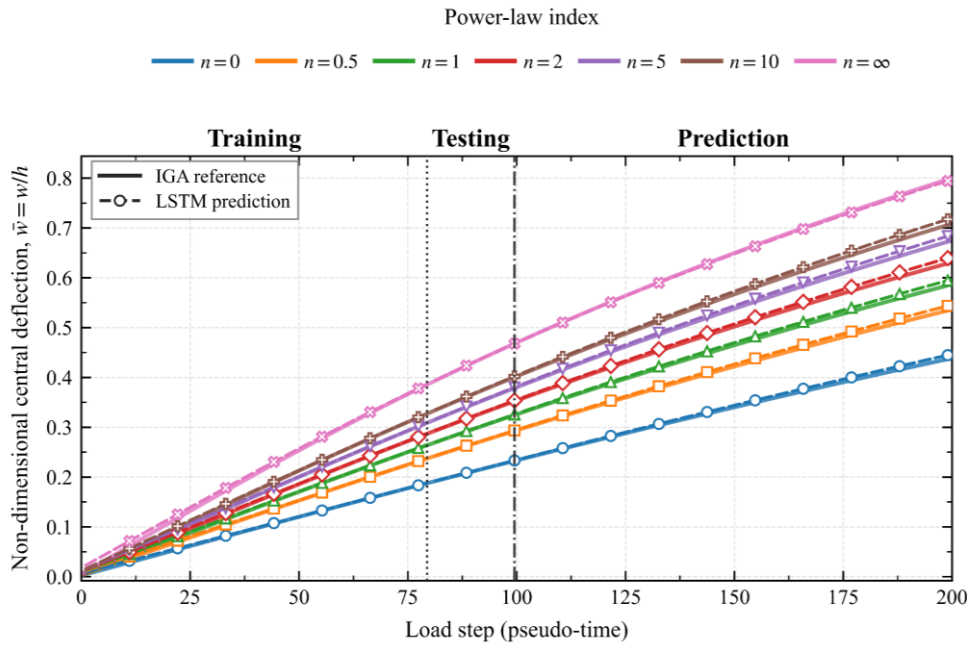


Fig. 4. LSTM predictions of the non-dimensional central deflection for different power-law indices, with 50% of the equilibrium-path sequence used for model development.

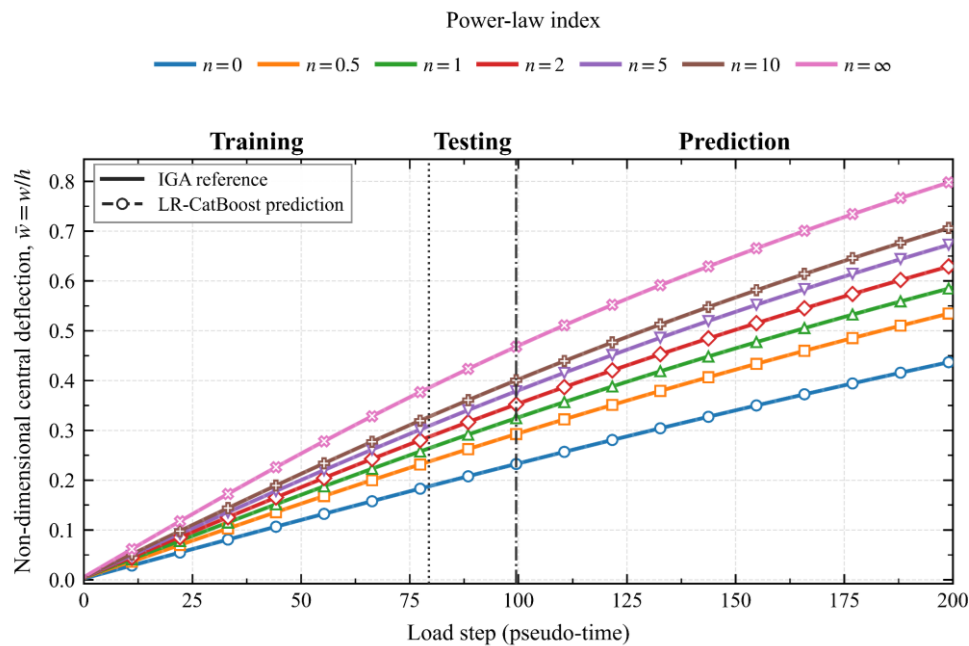


Fig. 5. LR-CatBoost predictions of the non-dimensional central deflection for different power-law indices, with 50% of the equilibrium-path sequence used for model development.

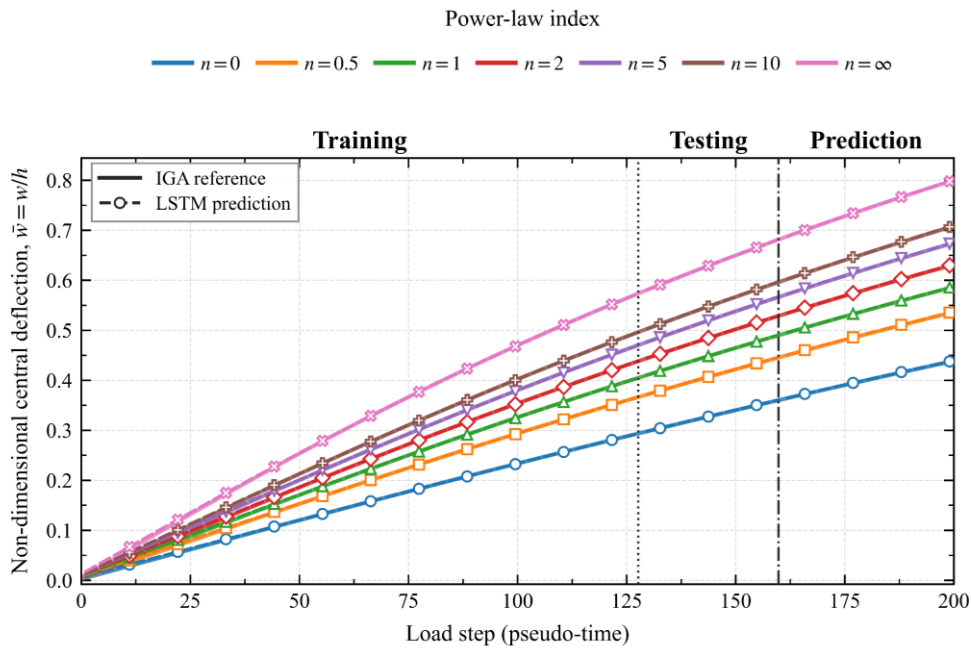


Fig. 6. LSTM predictions of the non-dimensional central deflection for different power-law indices, with 80% of the equilibrium-path sequence used for model development.

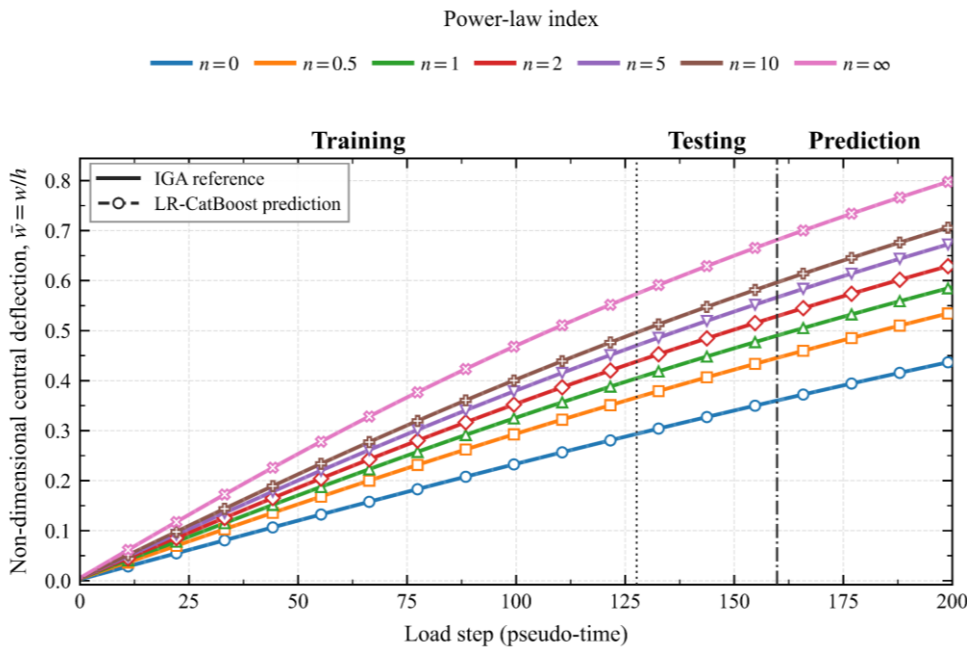


Fig. 7. LR-CatBoost predictions of the non-dimensional central deflection for different power-law indices, with 80% of the equilibrium-path sequence used for model development.

Figs. 4–7 provide a visual assessment of the predicted non-dimensional central deflection along the load-step sequence for the FG plate with the SSSS1 boundary condition under different power-law indices. The non-dimensional central deflection is defined as $\bar{w} = \frac{w}{h}$, where w denotes the transverse deflection at the plate center and h is

the plate thickness. Therefore, $\bar{w}$ is dimensionless and has no physical unit. In each panel, the full dataset is shown in blue, the training segment in red, the testing segment in green, and the model prediction in black. Fig. 4 and Fig. 5 correspond to the 50% data scenario, while Fig. 6 and Fig. 7 correspond to the 80% data scenario. In both cases, the predictions closely overlap the

reference response across the entire horizon, supporting the low error levels reported in Table 2. Comparing the two approaches, both the LSTM baseline and the proposed LR-CatBoost surrogate

deliver visually indistinguishable fits for this monotonic equilibrium-path sequence; however, LR-CatBoost achieves the same accuracy with a substantially lower end-to-end computational cost.

Table 3. Comparison between LSTM and LR-CatBoost for the FG plate (30% train+test).

Power index (n)	Method	Mean absolute percentage error (%)			Computation time (s)
		Training	Testing	Prediction	
0	LSTM	0.7590	0.2975	0.8207	352.95
	LR-CatBoost	0.0001	0.0047	0.0460	0.23
0.5	LSTM	0.1777	0.0935	0.5431	354.96
	LR-CatBoost	0.0002	0.0075	0.0664	0.23
1	LSTM	0.2926	0.1695	0.5724	349.77
	LR-CatBoost	0.0003	0.009	0.0757	0.26
2	LSTM	0.1223	0.1110	0.5637	361.43
	LR-CatBoost	0.0003	0.0101	0.0812	0.24
5	LSTM	0.2256	0.0499	0.4596	350.11
	LR-CatBoost	0.0003	0.0107	0.0844	0.24
10	LSTM	0.1139	0.0648	0.3612	348.25
	LR-CatBoost	0.0004	0.0117	0.0889	0.23
inf	LSTM	0.1538	0.0706	0.2177	353.49
	LR-CatBoost	0.0004	0.0160	0.1044	0.23

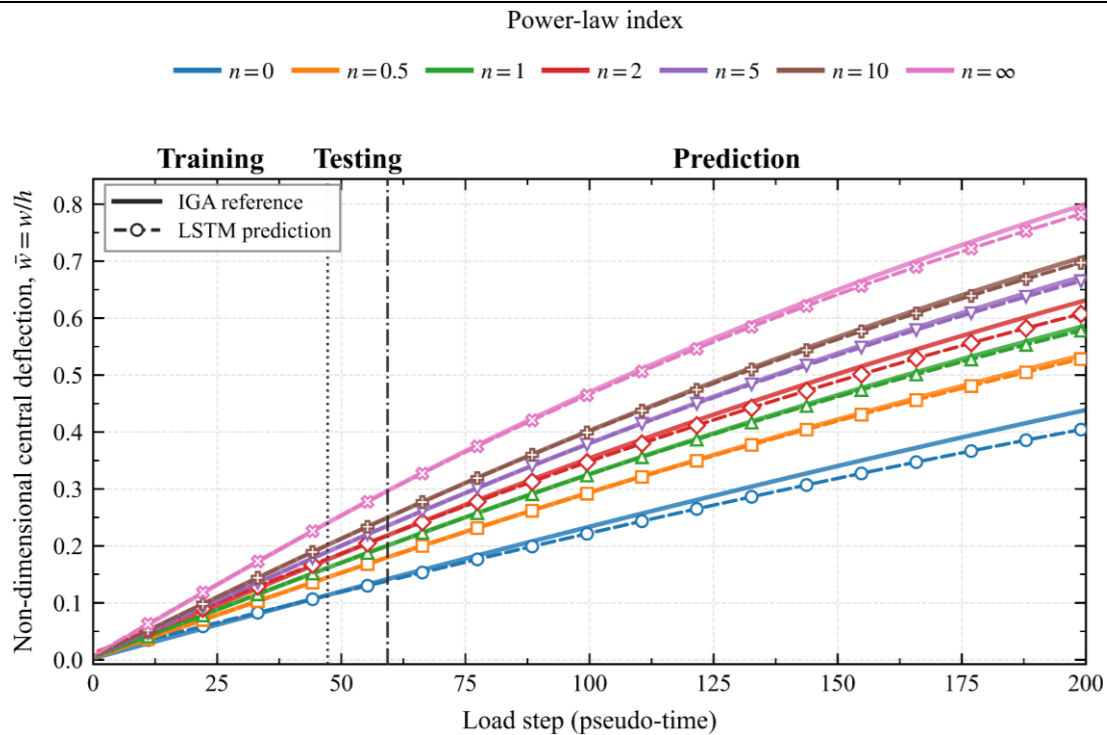


Fig. 8. LSTM predictions of the non-dimensional central deflection for different power-law indices, with 30% of the equilibrium-path sequence used for model development.

For the more data-scarce setting (30% of the sequence for model development), the LSTM

baseline typically requires a different set of hyperparameters/architecture (e.g., a deeper

network and more epochs) to achieve competitive accuracy, as also discussed in Ref. [18]. Therefore, LR-CatBoost was evaluated separately under the same 30% splitting rule to provide a consistent and lightweight alternative. The resulting errors and runtimes are summarized in Table 3. When only 30% of the sequence is available for training+testing, LR-CatBoost remains robust, with prediction MAPE around 0.046-0.104% and sub-second runtime (~ 0.23 - 0.26 s). In contrast, the LSTM baseline exhibits much larger forecasting errors and substantially higher cost (~ 350 s), and typically requires case-specific retuning to obtain competitive performance in this data-scarce

regime. For the more data-limited case (30%), Figs. 8 and 9 show the corresponding prediction profiles for LSTM and LR-CatBoost. Despite the reduced amount of training+testing data, both models still track the reference trajectory closely, with no obvious bias across the range of FGM indices.

Overall, the visual trends in Fig. 4 to Fig. 9 indicate that both learning strategies are robust for this benchmark response, and the choice between them can be guided by practical considerations such as computational cost, ease of tuning, and the expected complexity of temporal patterns in the target problem.

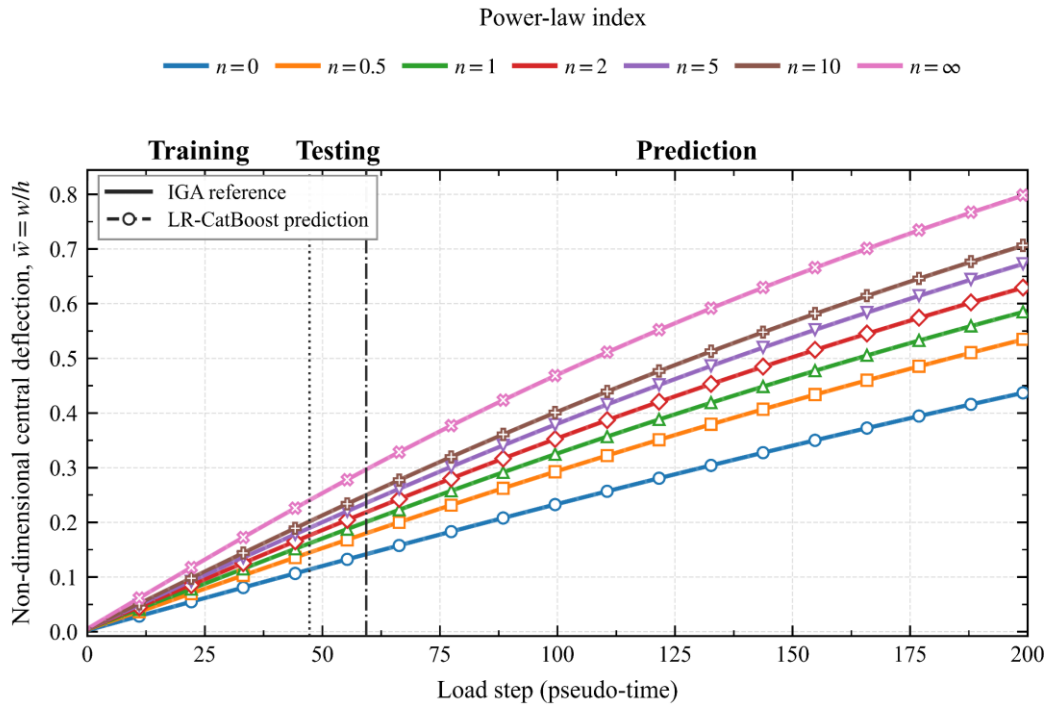


Fig. 9. LR-CatBoost predictions of the non-dimensional central deflection for different power-law indices, with 30% of the equilibrium-path sequence used for model development.

From a modeling perspective, LSTM learns the nonlinear sequence relationship directly and is capable of representing longer temporal dependencies when an appropriate look-back window, network architecture, and sufficient training data are provided. This flexibility may be beneficial for response histories with more complex temporal patterns; however, it is accompanied by higher training cost and greater sensitivity to architecture and hyperparameter selection. In

contrast, the proposed LR-CatBoost framework first extracts the dominant trend using a simple linear model and then assigns only the remaining nonlinear residual to CatBoost. This decomposition provides a more interpretable and computationally lightweight learning procedure and performs effectively for the smooth equilibrium paths and limited-data scenarios examined in this study. Nevertheless, the present $p=1$ formulation does not explicitly capture long-range dependencies,

and its performance for highly non-monotonic, multi-branch, or strongly path-dependent responses has not yet been established. The main novelty of the proposed method is therefore the use of a trend-residual LR-CatBoost architecture specifically for efficient equilibrium-path forecasting, rather than the direct learning of the complete response sequence.

Overall, the LR-CatBoost surrogate achieves consistently low forecasting errors across all investigated power-law indices and data splits while preserving a sub-second end-to-end runtime on a standard laptop. The results confirm that decomposing the equilibrium-path prediction into a linear trend component and a nonlinear residual learner yields both accuracy and efficiency, particularly when only a limited portion of the sequence is available for training. The main findings are summarized in the following conclusions.

5. Conclusions

This paper investigated a hybrid forecasting framework combining Linear Regression (LR) and CatBoost for sequence prediction of the nonlinear load-deflection equilibrium path in functionally graded (FG) plates. The proposed strategy leverages LR to represent the dominant global trend, while CatBoost is employed to learn the nonlinear residual component, resulting in an accurate and stable prediction pipeline. The model was systematically evaluated under different data-availability scenarios (30%, 50%, and 80% of the sequence used for training/testing) and across multiple material-parameter settings. The results demonstrate that the LR-CatBoost approach can reproduce the evolution of the response with high consistency across training, testing, and prediction intervals, while maintaining good generalization when the available data are limited. In particular, the model shows strong robustness in low-data cases (e.g., 30% and 50%), indicating that separating the linear and nonlinear components is effective for improving predictive reliability in

practical situations where only partial measurements are available. Compared with the LSTM-based model reported in our previous work, the current LR-CatBoost framework offers a competitive alternative with several practical advantages, including simpler training, reduced sensitivity to hyperparameter tuning, and significantly lower computational cost (sub-second runtime). These characteristics make LR-CatBoost a promising tool for efficient forecasting of FG structural responses in applications where computational efficiency and robustness are prioritized. Future work will extend the proposed model to a multivariate forecasting framework, consider broader boundary and loading conditions, and incorporate noise and uncertainty quantification to reflect measurement variability. Highly non-monotonic and path-dependent responses, such as snap-through, snap-back, and hysteresis, will also be investigated using longer look-back windows and additional state variables, together with transfer-learning or domain-adaptation strategies for improving generalization across different FG configurations.

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Declaration of Generative AI

During the preparation of this work, the authors used ChatGPT in order to improve the readability and language of the manuscript. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

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